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Velocities of pulsars and neutrino oscillations.

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Abstract

Neutrino oscillations, biased by the magnetic field, alter the shape of the neutrinosphere in a cooling protoneutron star emerging from the supernova collapse. The resulting anisotropy in the momentum of outgoing neutrinos can be the origin of the observed proper motions of pulsars. The connection between the pulsars velocities and neutrino oscillations results in a prediction for the τ neutrino mass of $m(\nu_\tau) \sim 100$ eV.

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It is well known that rotating magnetized neutron stars, pulsars, exhibit rapid proper motions [1, 2] characterized by space velocities that range in the hundreds of kilometers per second. Born in a supernova explosion, a pulsar may receive a substantial "kick" velocity due to the asymmetries in the collapse, explosion, and the neutrino emission affected by convection [3, 4]. Evolution of close binary systems may also produce rapidly moving pulsars [5]. Alternatively, it was argued [6] that the pulsar may be accelerated during the first few months after the supernova explosion by its electromagnetic radiation, the asymmetry resulting from the magnetic dipole moment being inclined to the rotation axis and offset from the center of the star. Most of these mechanisms, however, have difficulties explaining the magnitudes of pulsar spatial velocities, which can be as great as 1000 km/s and have an average value of 450 km/s [1]. (A recent study [4] shows, however, that a kick velocity of this magnitude can be achieved in an asymmetric collapse.)

In this letter we suggest an explanation for the birth velocities of the neutron stars based on asymmetric emission of neutrinos due to neutrino oscillations. We also show that the distribution of pulsar velocities can yield information about neutrino masses.

The basic idea is the following. Neutrinos emitted during the cooling of a protoneutron star have total momentum, roughly, 100 times the momentum of the proper motion of the pulsar. A 1% anisotropy in the neutrino distribution would result in a "kick" velocity consistent with observation. In the dense neutron star an electron neutrino, ν_e, has a shorter mean free path than ν_μ, ν_τ, or any of the antineutrinos. If one of the latter, *e. g.*, ν_τ, undergoes a resonant oscillation into ν_e, above the τ-neutrinosphere but below the e-neutrinosphere, it will be absorbed by the medium. Therefore, the *effective* τ-neutrinosphere in this case is determined by the point of resonance (Fig 1).

It is known that the electromagnetic properties of a neutrino propagating in medium, in a longitudinal magnetic field $\vec{B}$, are different from those in a vacuum [7]. The effective neutrino self-energy has a contribution proportional to $(\vec{B}\cdot\vec{k})$, where $\vec{k}$ is the neutrino momentum. The position of the resonance of the $\nu_\tau \to \nu_e$ oscillations is affected by the magnetic field and depends on the relative orientation of $\vec{B}$ and $\vec{k}$ [8]. Therefore the effective τ-neutrinosphere (or, "neutrinosurface", to be more precise) is not concentric with the electron neutrinosphere

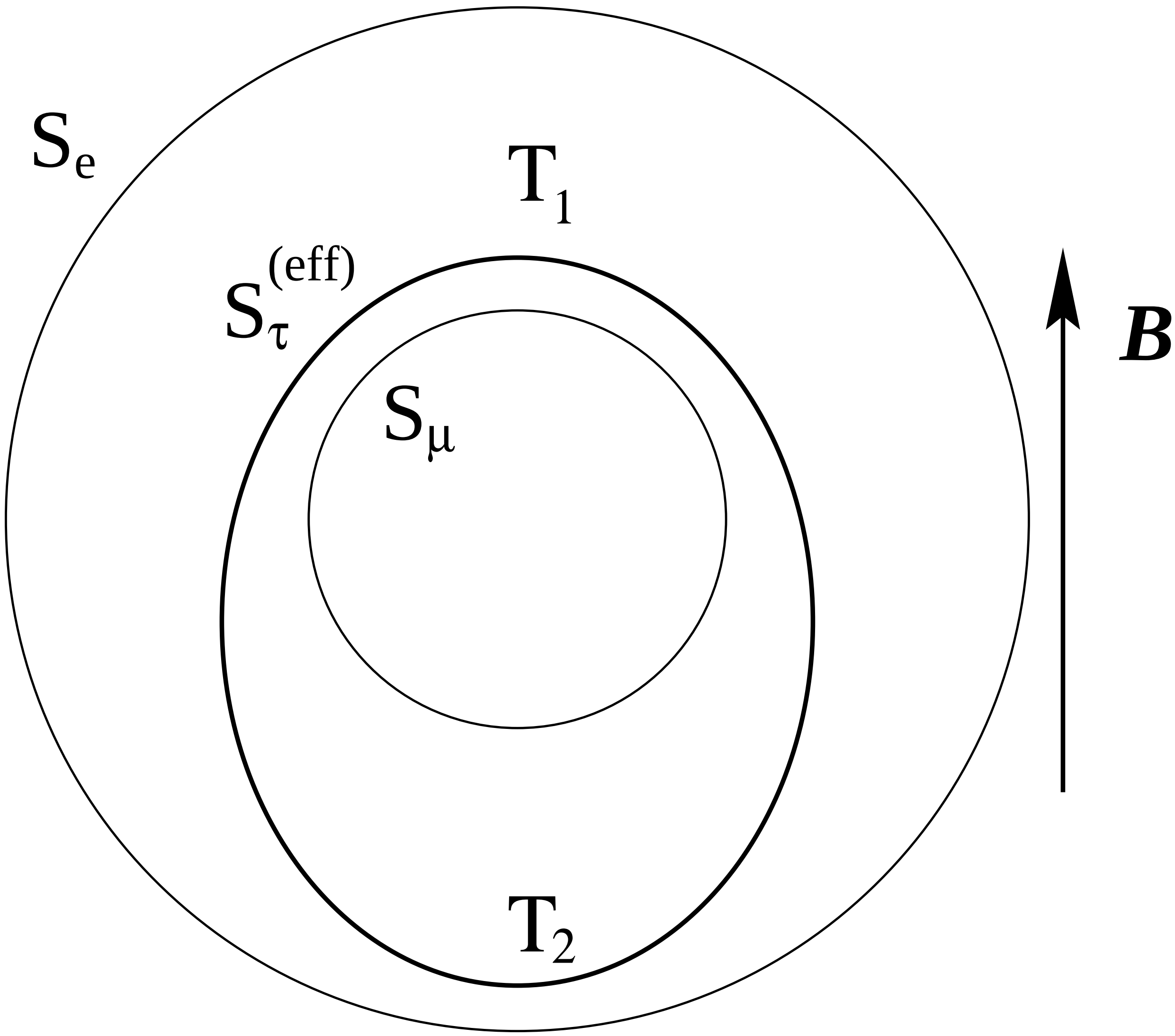

Figure 1: If the resonant oscillation $\nu_\tau \to \nu_e$ takes place between the electron neutrinosphere, S_e, and that of $\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$ and $\bar{\nu}_e$, S_μ, then the *effective* τ-neutrinosphere coincides with the surface of resonance, $S_\tau^{(\mathrm{eff})}$. The latter is affected by the magnetic field. Therefore, the τ-neutrinos emitted in different directions come from the regions of different temperatures. The resulting anisotropy in the momentum of the outgoing neutrinos can be the origin of the pulsar "kick" velocity, whose magnitude depends on the magnetic field.

(and, in fact, is not a sphere). The τ neutrinos that escape in the direction of magnetic field have, therefore, a different temperature from those emitted in the opposite direction. They carry away different momenta, thereby creating an asymmetry in good quantitative agreement with the data. We will show that this mechanism may be the origin of the proper motions of pulsars.

In a recent analysis of a sample of 99 pulsars, Lyne and Lorimer [1] concluded that the space velocity of pulsars at birth has a mean value of 450 ± 90 km s^{-1}. Typical pulsars have masses in the range from 1.0 $M_\odot$ to 1.5 $M_\odot$, where $M_\odot$ is the solar mass [9]. The momenta associated with the proper motion of pulsars are therefore of order $k_p \approx (1.2 \pm 0.4) \times 10^{41}\,\mathrm{g\,cm/s}$. The energy carried off by neutrinos in a supernova explosion is estimated to be $\sim 3 \times 10^{53}$ erg [10]. Since $p_\nu \approx E_\nu$ (we set $\hbar = c = 1$), this corresponds to the sum of the magnitudes of the neutrino momenta $\approx 10^{43}$ g cm/s. Comparing this with k_p, we conclude that a few per cent asymmetry in the distribution of the outgoing neutrinos is sufficient to give the pulsar a "kick" velocity ≈ 450 km s^{-1}.

The average energy of the neutrinos emitted during the cooling of the protoneutron star depends on the temperature of the neutrinosphere [10], defined as the surface where the optical depth, $\int_R^\infty dr/\lambda_\nu$, becomes of order 1. Neutrinos of different types have, in general, different neutrinospheres because of the difference in the mean free path, $\lambda_\nu \propto 1/\sigma_\nu$. Roughly speaking, one can consider the inside of the neutrinosphere to be opaque for neutrinos, while the outside is transparent. Electron neutrinos have both charged and neutral current interactions; they scatter off electrons, positrons and nucleons, as well as being absorbed [11] in the reaction $\nu_e n \to e^- p$. On the other hand, μ and τ neutrinos and antineutrinos can only scatter elastically, via neutral currents, at MeV temperatures. (The electron antineutrino, $\bar{\nu}_e$ can scatter and be absorbed in the process $\bar{\nu}_e p \to e^+ n$, so its opacity starts out close to that of ν_e and then decreases to, roughly, that of ν_μ and ν_τ as the density of protons decreases.)

This difference in scattering cross-sections gives rise to roughly an order of magnitude difference [11, 12] in the mean free paths of the ν_e and the ν_τ in the vicinity of the electron neutrinosphere, where the density is $10^{11} - 10^{12}$ g/cm^3. Consequently, the τ-neutrinosphere lies deeper inside the protoneutron star than the electron neutrinosphere. As a result, the τ

neutrinos escape from a deeper layer of the star, where the temperature is higher, and their mean energy exceeds that of ν_e's by about 50%.

We now consider neutrino oscillations. For definiteness, we discuss two flavors, ν_e and ν_τ, with $\Delta m^2 \equiv m^2(\nu_\tau) - m^2(\nu_e) \approx m^2(\nu_\tau) \gg m(\nu_e) \approx 0$ and small mixing. We will also assume that $\nu_e \to \nu_\mu$ oscillations take place in the Sun, while the transition $\nu_\tau \to \nu_e$ occurs at a much higher matter density. We leave a more careful analysis of the neutrino mass matrix consistent with all experimental data for future publication [13].

As was shown in Ref. [8], the neutrinos of energy $E \approx k = |\vec{k}|$ propagating in the degenerate electron gas of the charge density $N_e = n_{e^-} - n_{e^+}$, in magnetic field $\vec{B}$, undergo a resonant oscillation[1] if

$$\frac{\Delta m^2}{2k} \, cos\, 2\theta = \sqrt{2}\, G_{_F}\, N_e + \frac{eG_{_F}}{\sqrt{2}} \left(\frac{3N_e}{\pi^4}\right)^{1/3} \; \frac{\vec{k}\cdot\vec{B}}{k}, \tag{1}$$

where θ is the neutrino mixing angle in vacuum.

If the resonant conversion of ν_τ into ν_e occurs at some point between the τ neutrinosphere and the electron neutrinosphere, the τ neutrinos produced inside the "resonance-sphere" (*i. e.* closer to the center than the point of the resonance), will turn into ν_e's and will be absorbed (thermalized) by the medium. The τ-neutrinos produced by the $\nu_e \to \nu_\tau$ oscillations will escape with the energy determined by the temperature at the resonance point. Of course, τ-neutrinos produced outside the resonance-sphere will also escape, but there is no effective mechanism for copious production of τ-neutrinos outside neutrinosphere, where the matter density is small. The surface of resonance points for the $\nu_\tau \to \nu_e$ oscillations becomes the effective τ neutrinosphere. Since the last term in (1) depends on the relative orientation of $\vec{k}$ and $\vec{B}$, the resonant oscillations occur at different densities for the neutrinos going in different directions. The corresponding difference in temperatures will result in the asymmetry that gives the neutron star a "kick" in the direction of the magnetic field.

In the absence of the magnetic field, the resonance will occur at a distance r_0 from the

[1] We emphasize the difference between these chirality-preserving oscillations and the spin and flavor precession of neutrinos in magnetic field studied, *e. g.*, in Ref. [14]. The last term in equation (1) arises due to the effective interaction of neutrinos with the charged particles in the background and does not depend on the size of the neutrino magnetic dipole moment.

center if

$$\Delta m^2\, cos\, 2\theta = 2\sqrt{2}\, G_{_F}\, N_e(r_0)\, k. \tag{2}$$

If r_0 is to lie between the two neutrinospheres, where the density is $\approx 10^{11}$ g cm^{-3} and the electron to baryon ratio $Y_e \approx 0.1$, then, for a small mixing angle, Δm^2 has to be of order 10^4 eV^2, which corresponds to a τ neutrino mass of 100 eV, consistent with the current experimental limits [15]. This is close to the limit on stable neutrino masses imposed by cosmological constraints, $m_\nu < 92$ eV (Ωh_0^2) [16], although the mixed dark matter models favor a lower mass range [17] for stable neutrinos. Larger values of mass have been considered for unstable ν_τ's [18].

The oscillations are adiabatic as long as the density $N_e(r_0)$ and the magnetic field can be considered constant over the oscillation length, l_{osc},

$$l_{osc} \approx \left(\frac{1}{2\pi}\ \frac{\Delta m^2}{2k}\ sin\, 2\theta \right)^{-1} \approx \frac{1 \text{ cm}}{sin\, 2\theta}. \tag{3}$$

For a wide range of mixing angles, l_{osc} is much smaller than the scale on which the density changes are noticeable, so the oscillations can, in fact, be treated as adiabatic.

In the presence of the magnetic field, the condition (1) is satisfied at different distances from the center, depending on the value of the $(\vec{k} \cdot \vec{B})$ term in (1). The surface of the resonance is, therefore,

$$r(\phi) = r_0 + \delta\, cos\, \phi, \tag{4}$$

where $cos\, \phi = (\vec{k} \cdot \vec{B})/k$ and δ is determined by the equation:

$$2 \frac{dN_e(r)}{dr} \delta \approx e \left(\frac{3N_e}{\pi^4} \right)^{1/3} B. \tag{5}$$

This yields

$$\delta = \left(\frac{3N_e}{\pi^4} \right)^{1/3} \frac{e}{2}\, B \Bigg/ \frac{dN_e(r)}{dr} = \frac{e \mu_e}{2\pi^2}\, B \Bigg/ \frac{dN_e(r)}{dr}\ , \tag{6}$$

where $\mu_e \approx (3\pi^2 N_e)^{1/3}$ is the chemical potential of the degenerate (relativistic) electron gas.

We now estimate the size of the "kick" velocity from the effect just described. As was established in the beginning, a few per cent asymmetry in the momentum distribution of neutrinos is necessary to explain the observed pulsar velocities. Approximately 1/6 of the total energy is carried off by each of the neutrino species [10]. Integration over the angles in $(\vec{k} \cdot \vec{B})$ gives a factor 1/2 if the magnetic field is uniform. (The latter is, of course, a simplification.) The asymmetry in the third component of momentum is, therefore,

$$\frac{\Delta k}{k} = \frac{1}{6}\,\frac{1}{2}\,\frac{T^4(r_0-\delta) - T^4(r_0+\delta)}{T^4(r_0)} \approx \frac{2}{3}\,\frac{1}{T}\,\frac{dT}{dr}\,\delta, \tag{7}$$

where we have assumed a black-body radiation luminosity $\propto T^4$ for the effective neutrinosphere. Using the expression (6) for δ, we obtain

$$\frac{\Delta k}{k} = \frac{e}{3\pi^2}\left(\eta \frac{dT}{dN_e}\right) B, \tag{8}$$

where $\eta \equiv \mu_e/T$ is the degeneracy parameter of the electrons. Here we used the identity $(dT/dr)/(dN_e/dr) \equiv dT/dN_e$. To calculate the derivative in (8), we use the relation between the density and the temperature for the relativistic Fermi gas:

$$N_e = 2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(p-\mu)/T}+1}. \tag{9}$$

Differentiating the right-hand side with respect to T, we obtain $dN_e/dT \approx (2/3)\,T^2\,\eta$, for large η. (This asymptotic expression is accurate to three significant digits for $\eta > 5$.) We observe that for a degenerate relativistic electron gas ($\eta \gg 1$) the product $\eta(dT/dN_e) \approx 1.50/T^2$, independent of η. Finally, the ratio in (8) is

$$\frac{\Delta k}{k} = 0.015\,\frac{B}{T^2} = 0.01\left(\frac{3\ \mathrm{MeV}}{T}\right)^2 \left(\frac{B}{3\times 10^{14} G}\right). \tag{10}$$

During the cooling stage of the protoneutron star, the τ-neutrinos come out with the average energy ≈ 10 MeV [10], which corresponds to the temperature of ≈ 3 MeV. We see that the observed pulsar velocities, which require $\Delta k/k$ to be of order 0.01, can be explained by the values of $B \sim 3 \times 10^{14}$ G.

The magnetic fields at the surface of the neutron stars are estimated to be of order $10^{12}-10^{13}$G [9]. However, a magnetic field inside the pulsar may be as high as 10^{16}G [9, 19]. The existence of such a strong magnetic field is suggested by the dynamics of formation of the neutron stars, by the stability of the poloidal magnetic field outside the pulsar, as well as by the fact that the only star whose surface field is well studied, the Sun, has magnetic field below the surface which is 10^3 larger than that outside. (This strong magnetic field causes the sun spots when it penetrates the surface.)

Magnetic fields of order 10^{16}G inside the neutron star can themselves lead to asymmetric neutrino flux by the creation of neutron star analogue of sun spots. This phenomenon is disscussed in the context of strongly magnetized neutron starts by R. C. Duncan and C. Thompson [20]. In such strong magnetic fields, other weak interactions effects also become important [21]. We emphasize that the magnetic fields relevant for these effects are an order of magnitude higher than those considered in the present work.

It is clear from equation (10) that the deformations of the neutrinosphere due to neutrino oscillations biased by the magnetic field can result in the asymmetry of the neutrino flux necessary to give the pulsar a "kick" velocity consistent with the data.

In our calculations, we have assumed that the temperature distribution is not affected significantly by the position of the resonance. This assumption is well-justified because only the neutrinosphere of one out of six (anti-)neutrino species depends on the resonance, while the temperature profile is determined by the emission of all six. However, the increase of the heat drain in the vicinity of the effective τ-neutrinosphere makes the temperature gradient larger. This effect further increases the ratio (10). We have also neglected the deviations from thermal equilibrium in a cooling protoneutron star. A detailed analysis of the heat transport equations, though clearly important, is beyond the scope of this letter.

According to equation (10), the birth velocities of neutron stars are proportional to the magnetic field at the effective neutrinosphere. However, the direction of the proper motion of a pulsar need not be correlated with its angular momentum, because of the inclination of the magnetic field to the rotation axis and its possible offset from the center. This is also in good agreement with the observations which find no correlation between the space

velocities and the linear polarization of radio emission [22]. In a separate paper [23], we discuss the implications of our mechanism for the correlation between the magnetic fields and the magnitudes of pulsar velocities.

In conclusion, we have described a new explanation for the birth velocities of pulsars, which is in good agreement with the observational data. It presupposes, as we have stated, a τ neutrino mass of about 100 eV for small mixing. (A more complex realization of our mechanism may allow for smaller $m(\nu_\tau)$ [13].) The surprising connection between the proper motions of pulsars and neutrino oscillations provides a new astrophysical "laboratory" for neutrino physics. Measurements of the space velocities of pulsars can yield information about the neutrino masses in the range which is currently inaccessible to either solar neutrino, or collider experiments.

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